

B -parameters for $\Delta S = 2$ Supersymmetric Operators

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We present the first lattice measurement, using Non Perturbative Renormalization Method, of the B -parameters of the dimension-six four-fermion operators relevant for the supersymmetric corrections to the $\Delta S = 2$ transitions.

1. INTRODUCTION

This work is the first lattice calculation of the matrix elements of the most general set of $\Delta S = 2$ dimension-six four-fermion operators, renormalized non-perturbatively (NP) in the RI (MOM) scheme [1]–[5]. The main parameters and the details of the simulations are given in ref. [6]. Our results can be combined with the recent two-loop calculation of the anomalous dimension matrix in the same renormalization scheme [7] to obtain K^0 – \bar{K}^0 mixing amplitudes which are consistently computed at the next-to-leading order. A phenomenological application of the results for the matrix elements given below, combined with a complete next-to-leading order (NLO) evolution of the Wilson coefficients, can be found in [8].

The B -parameter of the matrix element $\langle \bar{K}^0 | O^{\Delta S=2} | K^0 \rangle$, commonly known as B_K , has been extensively studied on the lattice; for the other operators, instead, all the phenomenological analyses beyond the SM have used B -parameters equal to one, which in some cases is a very crude approximation. Moreover, with respect to other calculations, the systematic errors in our results are reduced by using the tree-level

improved Clover action [9,10] and by renormalizing NP the lattice operators.

We have used the supersymmetric basis ²:

$$\begin{aligned} O_1 &= \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) d^\alpha \bar{s}^\beta \gamma_\mu (1 - \gamma_5) d^\beta, \\ O_2 &= \bar{s}^\alpha (1 - \gamma_5) d^\alpha \bar{s}^\beta (1 - \gamma_5) d^\beta, \\ O_3 &= \bar{s}^\alpha (1 - \gamma_5) d^\beta \bar{s}^\beta (1 - \gamma_5) d^\alpha, \\ O_4 &= \bar{s}^\alpha (1 - \gamma_5) d^\alpha \bar{s}^\beta (1 + \gamma_5) d^\beta, \\ O_5 &= \bar{s}^\alpha (1 - \gamma_5) d^\beta \bar{s}^\beta (1 + \gamma_5) d^\alpha, \end{aligned} \quad (1)$$

where α and β are colour indices. The B -parameters for these operators are defined as

$$\begin{aligned} \langle \bar{K}^0 | \hat{O}_1(\mu) | K^0 \rangle &= \frac{8}{3} M_K^2 f_K^2 B_1(\mu), \\ \langle \bar{K}^0 | \hat{O}_2(\mu) | K^0 \rangle &= -\frac{5}{3} \frac{M_K^4 f_K^2}{(m_s(\mu) + m_d(\mu))^2} B_2(\mu), \\ \langle \bar{K}^0 | \hat{O}_3(\mu) | K^0 \rangle &= \frac{1}{3} \frac{M_K^4 f_K^2}{(m_s(\mu) + m_d(\mu))^2} B_3(\mu), \quad (2) \\ \langle \bar{K}^0 | \hat{O}_4(\mu) | K^0 \rangle &= 2 \frac{M_K^4 f_K^2}{(m_s(\mu) + m_d(\mu))^2} B_4(\mu), \\ \langle \bar{K}^0 | \hat{O}_5(\mu) | K^0 \rangle &= \frac{2}{3} \frac{M_K^4 f_K^2}{(m_s(\mu) + m_d(\mu))^2} B_5(\mu), \end{aligned}$$

²This basis is also the one for which the numerical values of the Wilson coefficients, computed at the NLO, have been given in [8].

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where the operators $\hat{O}_i(\mu)$ and the quark masses are renormalized at the scale μ in the same scheme. The numerical results for the B -parameters, $B_i(\mu)$, computed in this paper refer to the RI scheme.

2. NON-PERTURBATIVE RENORMALIZATION

Because of the breaking of the chiral symmetry with the Wilson fermion, each operator in the $\Delta S = 2$ Hamiltonian mixes with operators belonging to different chiral representations [11] so that the correct chiral behaviour is achieved only in the continuum limit. This represented a long-standing problem in the evaluation of B_K only recently solved with the introduction of Non-Perturbative Renormalization methods. In these approaches the renormalization constants (mixing matrix) are computed non-perturbatively on the lattice either by projecting on external quark and gluon states (NPM) as proposed in ref. [1] or by using chiral Ward Identities [12,13].

Recent studies of the B_K and of the B -parameters of the electro-penguin operators, $B_7^{3/2}$ and $B_8^{3/2}$ (which coincide with B_4 and B_5 respectively), with both non-perturbative renormalization methods, [2]–[4] and [13], found that the NP renormalization of the lattice operators gives B -parameters that significantly differ from those renormalized perturbatively (PT) [14]. The discretization effects are less important than those due to the PT evaluation of the mixing coefficients.

In this work we use the NPM renormalization. The results for all the renormalization constants for the complete basis of four-fermion operators (computed with the NPM, for several renormalization scales μ , at $\beta = 6.0$ and 6.2) can be found in [5].

3. B -PARAMETERS

The B -parameters are usually defined as

$$B_i(\mu) = \frac{\langle \bar{K}^0 | \hat{O}_i(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \hat{O}_i | K^0 \rangle_{VSA}}, \quad (3)$$

where VSA means Vacuum Saturation Approximation. The VSA values of the matrix elements of \hat{O}_4 and \hat{O}_5 differ from the factors appearing in the definition of the B -parameters in eq. (2) by the terms proportional to $1/3M_K^2 f_K^2$ and $M_K^2 f_K^2$ respectively. These terms, which originate from the squared matrix elements of the axial current, are of higher order in the chiral expansion and have been dropped in our definition of the B -parameters eq. (2). This implies that, out of the chiral limit, the values of B_4 and B_5 with our definition differ from those obtained by using (3). Out of the chiral limit, with the standard definition of the B -parameters obtained by using the VSA normalization, the scaling properties of $B_4(\mu)$ and $B_5(\mu)$ would have been much more complicated. The reason is that, in these cases, the VSA has a piece which scales as the squared pseudoscalar density and another one (proportional to the physical quantity $|\langle \bar{K}^0 | \hat{A}_\mu | 0 \rangle|^2$) which is renormalization group invariant. The μ -independence of the final result would then have been recovered in a very intricate way. Since the definition of the B -parameters is conventional, we prefer to use that of eq. (2), for which the scaling properties of all the B -parameters are the simplest ones. Moreover, with this choice, they are the same as those derived in the chiral limit.

We stress that $B_1 = B^{\Delta S=2}$, $B_4 = B_8^{3/2}$ and $B_5 = B_7^{3/2}$. In ref. [4], the results referred to the operators $O^{\Delta S=2}$, $O_8^{3/2}$ and $O_7^{3/2}$ at $\beta = 6.0$ only. In this paper, we present the results for all the B -parameters and for $\beta = 6.0$ and 6.2 .

4. NUMERICAL RESULTS

Our simulations have been performed at $\beta = 6.0$ and 6.2 with the tree-level Clover action, for several values of the quark masses, in the quenched approximation. In table 1, we summarize our results.

In constructing the renormalized operators we have used the central values of the renormalization constants neglecting their statistical errors.

At $\beta = 6.0$, the results for B_1 , B_4 and B_5 extrapolated to the chiral limit are slightly different from those of ref. [4]. There are several

	ref.[4]	this work				
	$\beta = 6.0$	$\beta = 6.0$		$\beta = 6.2$		best estimate
	$m_K = 0$	$m_K = 0$	$m_K = m_K^{exp}$	$m_K = 0$	$m_K = m_K^{exp}$	$m_K = m_K^{exp}$
$B_1 = B_K$	0.66(11)	0.70(15)	0.70(15)	0.68(21)	0.68(21)	0.69(21)
B_2	—	0.61(3)	0.66(3)	0.63(6)	0.66(4)	0.66(4)
B_3	—	1.10(8)	1.12(7)	0.94(16)	0.98(12)	1.05(12)
$B_4 = B_8^{3/2}$	1.03(3)	1.04(4)	1.05(3)	0.98 (8)	1.01(6)	1.03(6)
$B_5 = B_7^{3/2}$	0.72(5)	0.68(7)	0.79(6)	0.46(13)	0.67(10)	0.73(10)

Table 1

B -parameters at the renormalization scale $\mu = a^{-1} \simeq 2 \text{ GeV}$, corresponding to $\mu^2 a^2 = 0.96$ and $\mu^2 a^2 = 0.62$ at $\beta = 6.0$ and 6.2 respectively. All results are in the RI (MOM) scheme.

reasons for the differences: i) we fix the scale and the strange quark mass using the lattice-plane method of ref. [15]; ii) in the present analysis, we use the “lattice dispersion relation” instead than the continuum one used in [4]; iii) in order to reduce the systematic effects due to higher order terms in the chiral expansion, i.e. to higher powers of $p \cdot q$, we have not used the results corresponding to $\vec{p} = 2\pi/L(1, 0, 0)$ and $\vec{q} = 2\pi/L(-1, 0, 0)$. This choice stabilizes the results for B_1 between $\beta = 6.0$ and $\beta = 6.2$ whilst the results for the other B -parameters remain essentially unchanged.

5. CONCLUSIONS

Although we have data at two different values of the lattice spacing, the statistical errors, and the uncertainties in the extraction of the matrix elements, are too large to enable any extrapolation to the continuum limit $a \rightarrow 0$: within the precision of our results we cannot detect the dependence of B -parameters on a . For this reason, we estimate the central values by averaging the B -parameters obtained with the physical mass m_K^{exp} at the two values of β . Our best estimates are reported in the last column of the table 1. We observe that the lattice values of $B_{3,4}$ are close to their VSA whereas this is not true for $B_{1,2,5}$.

In ref. [14] B_2 and B_3 have been obtained at $\beta = 6.0$ with the Wilson action and the operators renormalized perturbatively in the $\overline{\text{MS}}$ scheme; the result is $B_2 = 0.59(1)$ and $B_3 = 0.79(1)$. Although a direct comparison is not possible (our results are in the RI scheme), to the extent that the

matching coefficients between the two schemes are a small effect [4], comparison of the NP results and the PT ones suggests that PT renormalization behaves poorly in some cases. This confirms the need for NP renormalization.

Our results allow an improvement in the accuracy of phenomenological analyses intended to put bounds on basic parameters of theories beyond the Standard Model.

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